A transient inverse problem in simultaneous estimation of TDTP based on MEGA

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Abstract

Purpose – The main aim of this paper is to utilize the different forms of functions for the numerical solution of the two-dimensional (2-D) inverse heat conduction problem with temperature-dependent thermo-physical properties (TDTPs).

Design/methodology/approach – The proposed numerical technique is based on the modified elitist genetic algorithm (MEGA) combined with finite different method (FDM) to simultaneously estimate temperature-dependent thermal conductivity and heat capacity. In this paper, simulated (noisy and filtered) temperatures are used instead of experimental data. The estimated temperatures are obtained from the direct numerical solution (FDM) of the 2-D conductive model by using an estimate for the unknown TDTPs and MEGA is used to minimize a least squares objective function containing estimated and simulated (noisy and filtered) temperatures.

Findings – The accuracy of the MEGA is assessed by comparing the estimated and the pre-selected TDTPs. The results show that the measurement errors do not considerably affect the accuracy of the estimates. In other words, the proposed method provides a practical and confident prediction in simultaneously estimating the temperature-dependent heat capacity and thermal conductivity. From the results, it is found that the RMS error between estimated and simulated temperatures is smaller for linear simulation and also we found this form convenient for parameters estimations.

Research limitations/implications – Future approaches should find the optimal design of case study and then apply the proposed method to achieve the best results.

Originality/value – Applications of the results presented in this paper can be of value in practical applications in parameter estimation even with one sensor temperature history.

Keywords Heat conduction, Data analysis, Thermodynamic properties

Paper type Research paper

Nomenclature

$C(T)$	Specific heat capacity
c_i	Individuals
$c_{best,i}$	The ith gene of the first-ranked individual
$f(\beta)$	Fitness function
I	Number of the time readings
$K(T)$	Thermal conductivity
L_i	Lower bounds of the ith gene of individuals
L	Width of slab
M	Number of sensors
n	Size of coded individuals or unknown parameters
P	Population
P_m	Mutation probability
P_r	Replacement probability
P_c	Crossover probability
q_c	Constant heat flux
r_c	Compression factor
$S(\beta)$	Least squares error
T_{exact}	Exact temperature
t_h	Heating duration
t_h^+	Non-dimensional heating duration

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t_f Total measurement time

t^+ Dimensionless temperature

T_{max} Maximum of the exact temperature

U_i Upper bounds of the ith gene of individuals

$\tilde{Y}_{im}$ Temperature histories

Greek symbols

α Average thermal diffusivity

β Estimated parameter vector containing the unknown TDTPs

β_j A gene

σ Standard deviation of the temperature measurements

ε Random variable

1. Introduction

Inverse heat conduction problems have received much attention since they have been widely used in practical engineering problems to estimate the thermal properties (Kim *et al.*, 2003, 2004) as well as the initial and boundary conditions (Raudenský, 1993; Chen *et al.*, 1997; Kim and Lee, 2002; Ranjbar and Mirsadeghi, 2007; Martorano and Capocchi, 2000). The aim of the present study is simultaneous estimation of temperature-dependent thermal conductivity and heat capacity which are, in general, dependent on the temperature. Inverse problems of estimating temperature-dependent thermo-physical properties (TDTPs) have been generally solved by using the conjugate gradient method with adjoint problem for parameter estimation or a common gradient-based method such as Gaussian linearization and modified Box-Kanemasu methods (Garcia, 1999; Imani *et al.*, 2006). In case there is no prior information about the functional form of unknown TDTPs or in the case of simultaneously estimating correlated parameters, such techniques are very difficult to apply because of their sensitivity to measurement errors (Imani *et al.*, 2006).

Beck and A1-Araji (1978) applied the simple transient method to estimate the specific heat, thermal diffusivity and contact conductance. In this work, the thermal conductivity is assumed to be constant or a linear function of temperature. Chen and Lin (1998) used a hybrid numerical algorithm of the Laplace transform technique and the control-volume method to simultaneously estimate the temperature-dependent thermal conductivity and heat capacity from temperature measurements inside the material. But, the functional forms of the thermal conductivity and heat capacity were unknown a priori. Huang and Ozisik (1991) applied a direct integration approach to estimate linear TDTPs. Their algorithm is not very sensitive to the choice of initial guesses, sensor location and experiment times, but needs curve-fitted measurements. Kim (2001) used a direct method to estimate the temperature-dependent thermal conductivity without internal measurements. He transformed the steady-state non-linear heat conduction equation into the Laplace equation via the Kirchhoff transformation. The thermal conductivity was modeled as a linear combination of known functions with unknown coefficients, which were directly determined from the imposed heat flux and measured temperatures at the boundary. Huang *et al.* (1995) and Huang and Yan (1995) applied the conjugate gradient method to estimate the temperature-dependent thermal conductivity $k(T)$ and heat capacity $c(T)$. At least two thermocouples were used for estimating $k(T)$ (Huang and Yan, 1995). Alifanov and Mikhailov (1978) applied the conjugate gradient method to search for the thermal conductivity. Tervola (1989) solved the problem through the Davidson-Fletcher-Powell method. Scarpa *et al.* (1993) found their solution via the Monte Carlo technique and covariance analysis. Such methods do not take into account measurement errors and are limited to linear cases. Genetic algorithm (GA) is a robust, non-gradient algorithm that belongs to the field of evolutionary algorithms. Garcia (1999) developed an excellent GA code to optimize the experiment design for estimation of TDTPs of composite materials. Garcia used this code in order to estimate directional K and C properties of composite. As his model was simple, he used analytical solution to find the temperature field. Imani *et al.* (2006) defined a simple model (1-D) and used one sensor to estimate simultaneously K and C based on TDTPs.

The aim of the present work is estimation of conductivity and heat capacity based on GA. Present model is non-linear case and there is no analytical solution for this model. In this study, six sensors are used and also the defined model is a practical and two-dimensional (2-D) one with appropriate boundary conditions. Finite different method (FDM) is used to calculate the temperature field. Moreover, a comparison between different estimation forms (constant, linear and parabolic function) is presented.

2. Direct problem

Consider a 2-D homogeneous slab as shown in Figure 1 used in engineering application (Kim *et al.*, 2003; Venkatesan *et al.*, 2001). The thermal conductivity $K(T)$ and specific heat capacity $C(T)$ are unknown. The boundary surface at $x = 0$ is subjected to a prescribed constant heat flux q_c during particular time as t_h. The other boundary surfaces are kept insulated. The governing differential equation of this problem can be expressed as:

$$\frac{\partial}{\partial x}\left[K(T)\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[K(T)\frac{\partial T}{\partial y}\right] = \rho C(T)\frac{\partial T}{\partial t} \quad \text{for} \left| \begin{array}{l} 0 < x < L \\ 0 < y < 2L \end{array} \right. \quad \text{and} \quad |t > 0 \tag{2.1}$$

and boundary conditions are in the following forms:

$$K(T)\left.\frac{\partial T}{\partial x}\right|_{x=0} = f(t,y), \qquad \left| \begin{array}{ll} f(t,y) = q_c, & (L < y < 2L) \wedge (t < t_h) \\ f(t,y) = 0, & (0 < y < L) \vee (t > t_h) \end{array} \right. \tag{2.2a}$$

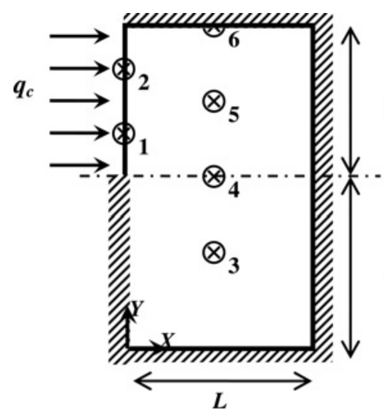


Figure 1.
2-D conductive model

$$\begin{cases} K(T)\dfrac{\partial T}{\partial x}\Big|_{x=L} = 0 \\ K(T)\dfrac{\partial T}{\partial x}\Big|_{y=0} = 0 \ , \quad t > 0 \\ K(T)\dfrac{\partial T}{\partial x}\Big|_{y=2L} = 0 \end{cases} \tag{2.2b}$$

where t_h is heating duration and constant heat flux $q_c = 120\,\mathrm{kW\,m^{-2}}$ is applied at the left boundary. The boundary conditions have also been shown in Figure 1 and the initial condition is:

$$T(x,y,0) = T_{\mathrm{init}} \tag{2.3}$$

Dimensionless temperature, time and heating duration are defined as:

$$T^{+}(x,y,t) = \frac{T(x,y,t)}{q_c L/K}, \quad t^{+} = \frac{\alpha t}{L^2}, \quad t_h^{+} = \frac{t_h}{t_f} \tag{2.4}$$

where α is the average thermal diffusivity of the sample, t_h is the heating duration and t_f is the total measurement time. In this study, $L_x = 0.1\,\mathrm{m}$, $K = 14.1\,\mathrm{W\,m^{-1}\,K^{-1}}$, $C_p = 448\,\mathrm{J\,kg^{-1}\,K^{-1}}$, $\rho = 6{,}288.25\,\mathrm{kg\,m^{-3}}$ and $T_{\mathrm{init}} = 23°\mathrm{C}$ are considered and used in order to define the dimensionless variables.

For the direct heat conduction problem, the temperature distribution in the slab as a function of space and time can be numerically (FDM) determined (Gerald and Wheatley, 1997) provided that all TDTPs of the slab are given.

3. Measurement simulation

In order to simulate the experiment, the temperature histories at some sensor locations are usually measured in the slab. We use the temperature histories taken from the sensors and denoted them by $\tilde{Y}_{im}$ (t_i, $Sensor_m$) $\equiv \tilde{Y}_{im}$, $i = 1$ to I and $m = 1$ to M, where I denotes the number of the time readings and M is the number of sensors. We used six sensors in this study ($I = 6$). The sensor locations have been shown in Figure 1 and also all the exact position of each sensor is clear in Table I. The measured temperature data, $\tilde{Y}_{im}$, used in the present inverse analysis can be determined from the exact temperature solution of the direct heat conduction problem with the given TDTPs, T_{exact}. Owing to experimental uncertainty, $\tilde{Y}_{im}$ contains the measurement error. Thus, T_{exact} should be modified by Gaussian additive noise in order to simulate experimental measurements. With respect to the eight statistical assumptions in Beck

Table I.
X- and *Y*-positions for sensors

Sensor number	*X*-position	*Y*-position
1	0	2*L*-2*L*/3
2	0	2*L*-*L*/3
3	*L*/2	*L*-*L*/2
4	*L*/2	*L*
5	*L*/2	2*L*-*L*/2
6	*L*/2	2*L*

et al. (1985) and Ozisik and Orlande (2000), $\tilde{Y}_{im}$ can be expressed as:

$$\tilde{Y}_{im} = T_{exact}(t_i, Sensor_m) + \varepsilon\sigma \tag{3.1}$$

where σ is the standard deviation of the temperature measurements and ε is a random variable ranging from -2.576 to 2.576 for normally distributed errors with zero mean and 99 per cent confidence bounds. The product of $\varepsilon\sigma$ represents the temperature measurement error.

4. Inverse analysis

Inverse parameter estimation methods are based on the minimization of an objective function containing both estimated and measured temperatures (Beck *et al.*, 1985; Beck and Arnold, 1977). Ordinary least squares (OLS) estimator is by far the most frequently used method for the estimation of TDTPs as no prior knowledge is needed (Beck and Arnold, 1977). OLS estimator was considered in this research. The associated objective function, the least squares error, S, is expressed by (Ozisik and Orlande, 2000):

$$S(\beta) = \sum_{m=1}^{M}\sum_{i=1}^{I}[\tilde{Y}_{im} - T_{im}(\beta)]^2 \tag{4.1}$$

where β is the estimated parameter vector containing the unknown TDTPs; $\tilde{Y}_{im}$ is the ith observation from the mth sensor; M and I are the number of sensors and observations, respectively. $T_{im}(\beta)$ is the calculated temperature from the mathematical model governing (direct solution) the heat transfer phenomena with respect to the estimated parameter vector.

In using Equation (4.1), the thermal properties are found by minimizing the sum of squared differences between the measured and calculated data. The minimization of Equation (4.1) could conceivably be performed by any optimization technique. However, parameter estimation has generally been performed with only a few methods. The use of one method over another is often specific to a certain field of study. The approach investigated in the present work involves the use of a robust non-gradient method, namely the GA method, in the minimization procedure. The motivation for using GAs was to circumvent difficulties of non-convergence in cases when the parameters are correlated or nearly so.

5. Genetic algorithm

GAs were developed by Holland (Goldberg, 1989). The common feature of these algorithms is to simulate the search process of natural evolution and take advantage of the Darwinian survival-of-the-fittest principle. In short, evolutionary algorithms start with an arbitrarily initialized population of coded individuals with size n_s, in which a population P consists of individuals, c_i with $i = 1$ to n_s:

$$P = \{c_1, c_2, \ldots, c_{ns}\} \tag{5.1}$$

The population evolves toward increasingly better regions of the search space by means of both random and probabilistic methods (or deterministic methods in some algorithms). An individual is a possible solution of an optimization problem with the objective function $S(\beta)$, which is a scalar-valued function of an n-dimensional vector β.

The vector β consists of n unknown parameters β_j, with $j = 1$ to n, which represent a point in real space R^n. The variable β_j is called a gene. Thus, an individual c_i consists of n genes:

$$c_i = \{c_{i1}, c_{i2}, \ldots, c_{in}\} \tag{5.2}$$

The goodness of each individual is evaluated by a fitness function that is defined from the objective function of the optimization problem. To define a fitness function for minimization problems such as Equation (4.1), it is necessary to change the objective function, because GA works according to the principle of the maximization of the fitness function, and so the fitness function of Equation (4.1) is defined as:

$$f(\beta) = \frac{1}{0.001 + \sqrt{S(\beta)}} \tag{5.3}$$

The square root function is included to moderate the selection pressure of the GA, and 0.001 is added arbitrarily to limit the maximum of the fitness function and avoid the infinity.

The basic operators used in GAs consist of selection (the selection of parents for breeding), crossover (the exchange of parental genes to create children) and mutation (the changing of individual genes) (Michalewicz, 1996). The present mechanism to select parents is the combination of roulette wheel selection and tournament selection, where each individual in the current population has a roulette wheel slot sized in proportion to its fitness. An arithmetic crossover (Doyle, 1995) is applied to each pair of the mating pool with a crossover probability, P_c. A mutation operator modifies gene values of individuals according to a mutation probability, P_m. In addition, following the Darwinian Theory, an elitism operator (the protection of best individuals), e.g. generational replacement with probability P_r is used, in which parents are replaced with children, while the $n_s \times (1 - P_r)$ best parents are kept (Goldberg, 1989).

Goldberg (1989) developed the basic elitist genetic algorithm (BEGA). In this work, BEGA is modified with some additional operators such as an elite initial population and a domain compression operator (Imani *et al.*, 2006). The modified elitist genetic algorithm (MEGA) started by a successive random search for elite individuals in which only the first-ranked individual of each initial population is kept for an elite initial population. A compression factor r_c is then applied in some generations to reduce the parameters' search space as follows:

$$\begin{aligned} U_i|_{new} &= (1 - r_c)c_{best,i} + r_c U_i|_{old} \\ L_i|_{new} &= (1 - r_c)c_{best,i} + r_c L_i|_{old} \end{aligned} \tag{5.4}$$

where U_i and L_i are upper and lower bounds of the ith gene of individuals, respectively. And $c_{best,i}$ is the ith gene of the first-ranked individual in the generation in which the domain compression operator is applied.

There are many advantages of applying GAs to estimation problems. GAs are easily programmed. Their major strength is that they are derivative-free calculations and, as shown in this work, they do not need any initial guesses. Design of robust GAs is highly application specific and their performance is difficult to predict. Another significant drawback is the high CPU cost. A mathematical function called *f*6 (Ranjbar and Mirsadeghi, 2007; Imani *et al.*, 2006; Davis, 1991; Schaffer *et al.*, 1989) was optimized to illustrate the performance of the MEGA. The expression of this function is:

$$f6(x,y) = 0.5 - \frac{\left[\sin\sqrt{x^2+y^2}\right]^2 - 0.5}{\left[1.0 + 0.01 \times (x^2+y^2)\right]^2} \tag{5.5}$$

The goal is to optimize $f6$, e.g. to find values of x and y that produce the greatest possible value for $f6$. This function has some interesting features such as a single global optimum, which is $f6(x=0, y=0) = 1$, strong oscillations, and a tiny fraction of the total area for the global regions. Figure 2 shows a typical increase of both the fitness (function $f6$) of the best individual and the average fitness of the population obtained from MEGA for different runs. Accuracy of the present algorithm has been shown in Table II. Present algorithm successfully finds the global optimum just with 100 generations whereby x and y errors are from -4 to -8 order.

6. Methodology

A flowchart of the proposed method for TDTPs estimation is shown in Figure 3. A simulated experiment was performed with adding Gaussian white noise to the exact solution of the direct conductive model. To simulate the experiment, the pre-selected TDTPs are assumed for three cases of dependency as:

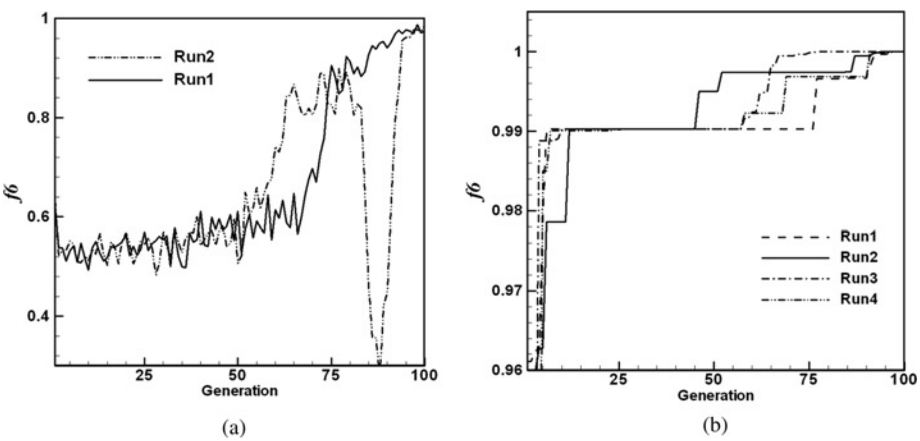


Figure 2. Average (a) and the best fitness (b) evolution of function $f6$ for different runs

Run	Best x	Best y	Best $f6$ fitness	Average fitness
1	−3.878E-06	3.628E-06	0.99999999997	0.9868
2	1.421E-06	−6.981E-07	1.00000000000	0.9834
3	−1.885E-04	−2.571E-05	0.99999996377	0.9727
4	5.842E-05	−6.015E-05	0.99999999296	0.9847
5	4.620E-05	−2.734E-06	0.99999999786	0.9841
6	−5.359E-07	4.348E-08	1.00000000000	0.9729

Notes: $n_s = 100$, $n_g = 100$, $P_c = 0.99$, $P_m = 0.1$, $P_r = 0.95$ and $r_c = 0.95$

Table II. Best x, y, fitness and average evolution of function f_6 for different runs with genetic parameters

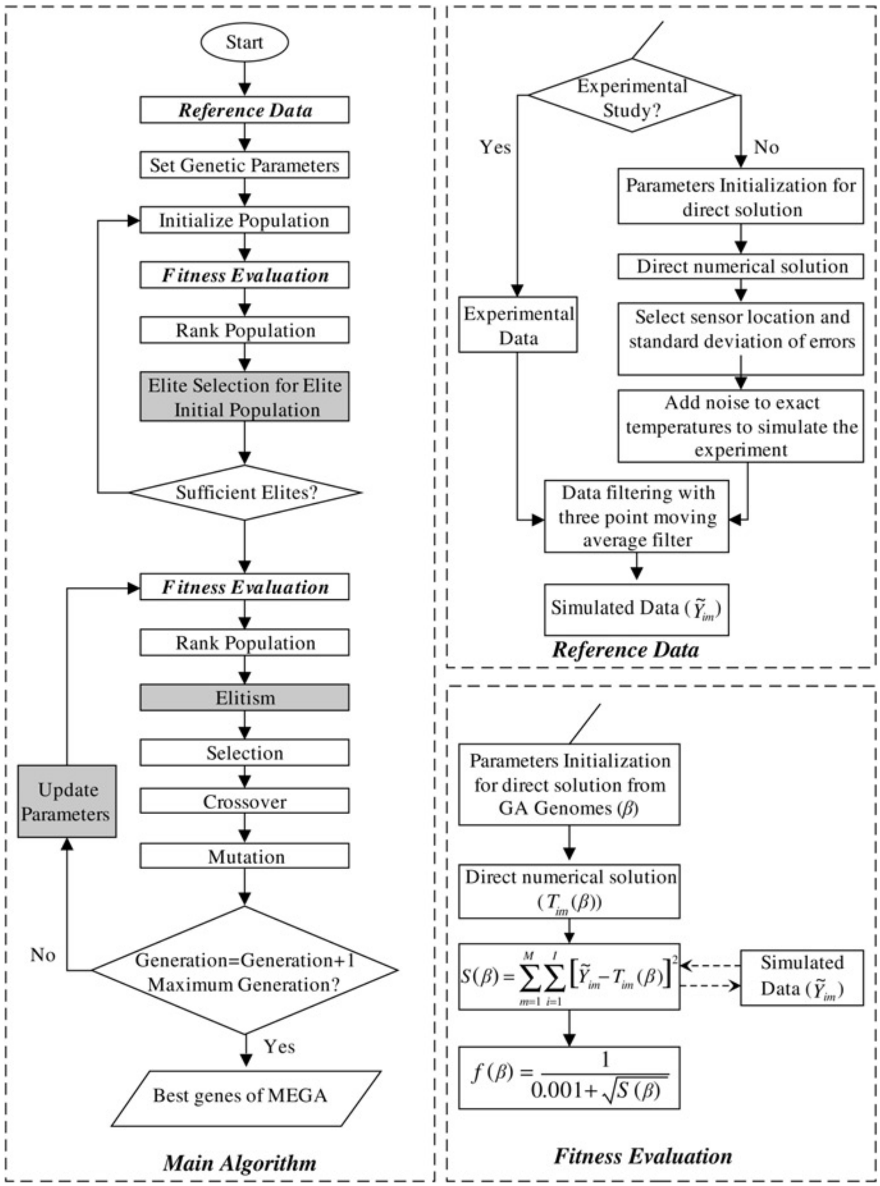


Figure 3.
Flowchart of the proposed method

$$
\begin{aligned}
K(T) &= k_1 = 14.1 \\
C(T) &= c_1 = 448
\end{aligned}
\tag{6.1}
$$

$$
\begin{aligned}
K(T) &= k_1 + k_2 T = 14.1 + 0.0166T \\
C(T) &= c_1 + c_2 T = 448 + 0.291T
\end{aligned}
\tag{6.2}
$$

$$K(T) = k_1 + k_2T + k_3T^2 = 14.1 + 0.0200T - 0.000021T^2$$
$$C(T) = c_1 + c_2T + c_3T^2 = 448 + 0.5455T - 0.0005597T^2 \quad (6.3)$$

A total of 500 simulated measurements containing additive, uncorrelated and normally distributed errors with zero mean and constant standard deviation of $\sigma = 0.01T_{max}$ were assumed available for the estimation procedure, where T_{max} is the maximum of the exact temperature in the simulated experiment. Measurement interval is chosen as 4 s. regarding boundary conditions and pre-selected TDTPs (Equations (6.1)-(6.3)), T_{max} is calculated as 403°C.

Exact simulated temperature measurements for all sensors locations are presented in Figure 4(a) for heating duration $t_h^+ = 0.3$. Actual measured data could be used for the inverse analysis as illustrated in Figure 3. A three-point moving average filter is applied to reduce measurement errors. After completion of data filtering, these measurements are used in MEGA to estimate the unknown TDTPs. Filtered temperatures for sensor Nos. 1 and 6 are compared with noisy and exact measurements in Figure 4(b). It is clear in Figure 4(b) that the data filtering reduces the noises at temperature history; this point is obviously shown in Figure 4(c).

7. Results and discussion

Genetic parameters could affect the convergence and performance of the MEGA. There are unfortunately few heuristics to guide a user in the selection of appropriate operators and genetic parameter settings for a particular problem. What can be grasped from the literature is that good GA performance requires the choice of a moderate population size, a high crossover probability and a low mutation probability (Garcia, 1999). So, genetic parameters in the current research are chosen as: $n_s = 100$; $n_g = 1{,}000$; $P_c = 0.99$; $P_m = 0.1$; $P_r = 0.95$; $r_c = 0.9,\ 0.95,\ 0.98$ when $n = 2,\ 4,\ 6$, respectively. The valid ranges for the unknown parameters are specified to begin the MEGA search, e.g. $c \in$ (0-1,000) and $k \in$ (0-100). The ranges of the second and third parameters in Equations (6.1)-(6.3) are obtained from the functional form of the unknown TDTPs considering the above valid ranges.

In this work, it is assumed that the form of the estimated TDTPs is priori unknown as in most real-world engineering applications. One may choose different forms of the unknown estimated TDTPs and calculate the RMS error between simulated and estimated temperatures to find minimum RMS error (maximum fitness function). e.g. in order to find linear form of TDTPs, the problem is to find the best combination of parameters (k_1, k_2, c_1, c_2) to achieve the maximum value of fitness function (minimum of RMS); therefore GA finds the best combination of parameters as β vector ($\beta \equiv [k_1, k_2, c_1, c_2]$) to gain the $f(\beta)$ maximum.

The inverse estimation of TDTPs is first performed by assuming exact measurements. Then, analysis is repeated for the noisy measurements with $\sigma = 0.01T_{max}$ for constant, linear and parabolic TDTPs. As randomness plays an important role in each run of the MEGA (two runs with different random seeds will generally produce different output). Figure 5 shows the differences between exact ($\sigma = 0$) and noisy ($\sigma = 0.01T_{max}$) measurements regarding the fitness function. Fitness functions for both cases are shown in the figures when one of the parameters changes around the pre-selected value of the parameter (e.g. $\beta_{;1} \in k_1 \pm 0.05 \times k_1$, Figure 5(a)) and the other parameters hold exactly the pre-selected value from the parabolic estimation (Equation (6.3)) (e.g. $\beta_i = k_i$ or c_i). Actually, β_1 stays on the x-axis

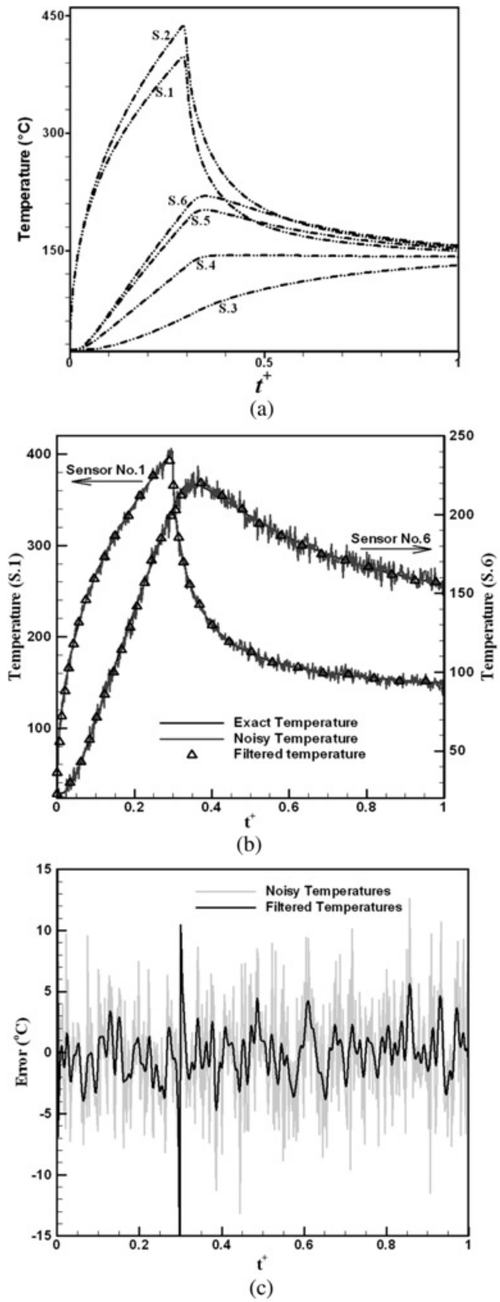
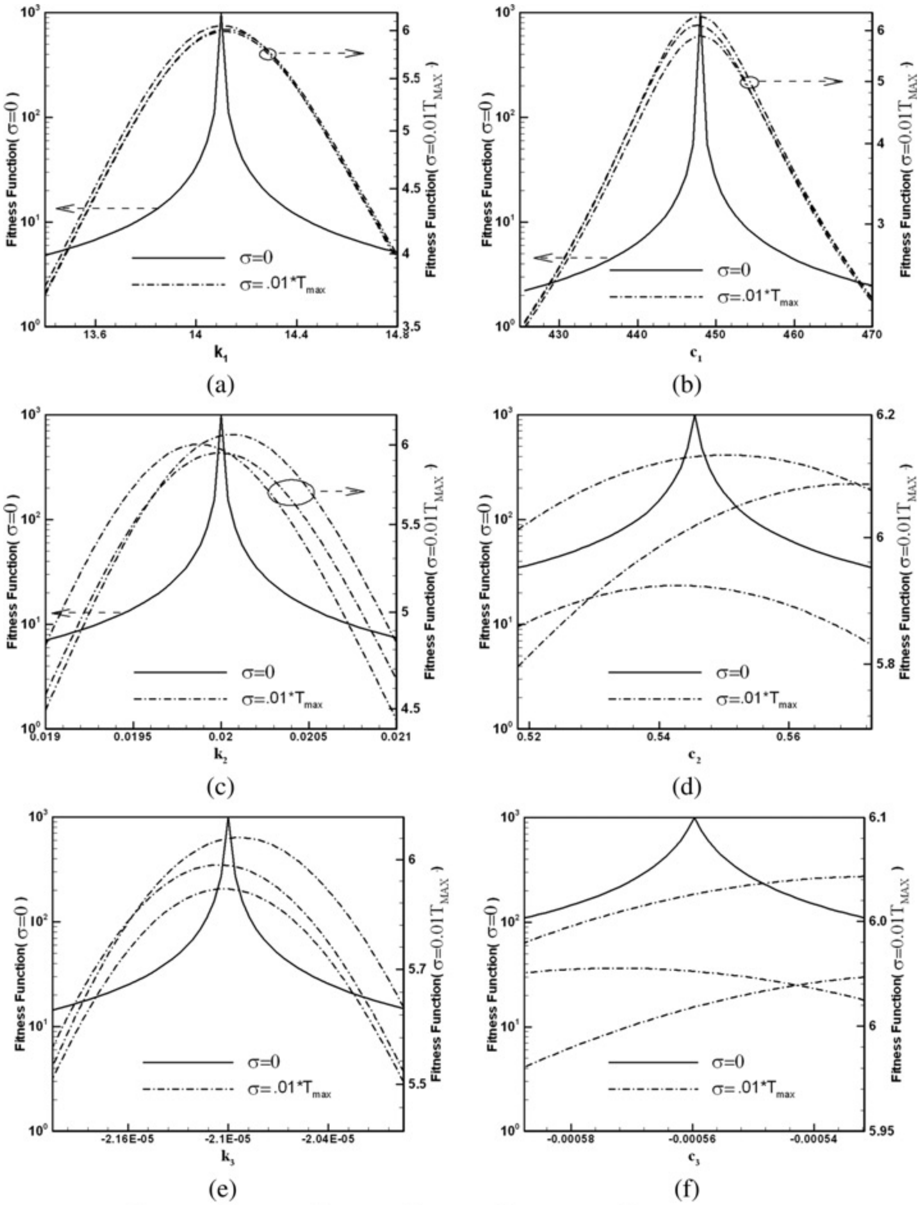


Figure 4.

Notes: (a) Exact simulated measurements of temperature at internal locations for $t_h^+ = 0.3$; (b) exact, noisy and filtered temperatures with $\sigma = 0.01T_{max}$ for sensor Nos 1 and 6; (c) error comparison between noisy and filtered temperature based on exact temperature of sensor No. 6



Notes: (a) $f(\beta_1)$, (b) $f(\beta_2)$, (c) $f(\beta_3)$, (d) $f(\beta_4)$, (e) $f(\beta_5)$ and (f) $f(\beta_6)$

Figure 5.
Comparison of fitness functions for $\sigma = 0$ and $\sigma = 0.01T_{max}$ for conductivity ($\beta_{1,3,5}$ or $k_{1,2,3}$) and heat capacity ($\beta_{2,4,6}$ or $c_{1,2,3}$) parameters for different runs

and vary from $\beta_1 = (1 - 0.05) \times k_1$ to $\beta_1 = (1 + 0.05) \times k_1$ and it is the difference between the cases in Figure 5 and the best fitness sit on the *y*-axis. In order to make it clear, it should be mentioned that $\beta_2 \equiv c_1 \pm 0.05 \times c_1$, $\beta_3 \equiv k_2 \pm 0.05 \times k_2$, $\beta_4 \equiv c_2 \pm 0.05 \times c_2$, $\beta_5 \equiv k_3 \pm 0.05 \times k_3$ and $\beta_6 \equiv c_3 \pm 0.05 \times c_3$ correspond Figures 5(b), (c), (d), (e) and (f) respectively. Moreover, each figure has one solid and three dashed-dot lines. The solid one corresponds to the $\sigma = 0$ case, while the dashed-dot lines are from different runs (random temperature error measurements) with $\sigma = 0.01T_{max}$. As demonstrated in the legend the solid line corresponds to the left hand y-axis and the other dashed-dot lines correspond to the right hand y-axis.

Generally speaking, for the exact measurements case, fitness functions reach the maximum value (1,000) as a result of negligible error between the exact measurements and direct solution. But for the noisy measurements, fitness functions do not exceed 6.2 due to noises at simulated temperature. Another point came from Figure 5 is that the optimum parameter to achieve the maximum fitness function value, does not fit on the pre-selected value and the optimum point changes a little in different runs due to noisy random temperature. This point is obvious in Figures 5(c), (d) and (f). The curves slip and differential range of fitness function show the sensitivity of each parameter as it is clear that the β_6 (or c_3) plays the least role in fitness function (Figure 5(f)).

Based on exact measurements three different runs are presented in Table III. We have started with exact simulated measurements and tried to find the best parameters with the same function used in simulation; two ($\beta \equiv [k_1, c_1]$), four ($\beta \equiv [k_1, k_2, c_1, c_2]$) and six ($\beta \equiv [k_1, k_2, k_3, c_1, c_2, c_3]$) parameters searching for constant, linear and parabolic simulation, respectively. The best and average fitness functions for the runs listed in Table III are also shown in Figures 6(a), (c) and (e). In the exact measurements case, as it mentioned, due to negligible error between the exact measurements and direct solution, we can access to value 1,000 of fitness function. It is clear in Figure 6(a) that in two parameters searching (constant simulation), the maximum value of fitness has been achieved in about the 60th generation, and this range is 700 for four parameters searching. For the six parameters case, the present algorithm could not meet the maximum fitness function (1,000) by 1,000 generations due to larger compression factor than the others ($r_c = 0.99$ when $n = 6$).

The results that are obtained from the exact measurements ($\sigma = 0$) in Table III show an excellent agreement between estimates and pre-selected TDTPs. As we can see, the RMS error between simulated and estimated temperatures is very small even for parabolic case (equal to 1.E-16 to 1.E-5). The details of these data are available in Table III for different runs. For noisy measurement, five different runs have been done and the results for linear simulation and estimation are shown in Table IV for instance. The good agreement has been observed between the estimation and pre-selected parameter.

The same procedure has been done for other simulation and estimation function types and the estimated parameters and RMS errors have been shown in Table V. The best and average fitness functions for each type of simulation or estimation function have been shown in Figures 6(b), (d) and (f). The constant estimation type was performed with constant, linear and parabolic simulation function and average and the best fitness function have been shown in Figure 6(b). The same has been done for linear and parabolic estimation and are shown in Figures 6(d) and (f), respectively. The mean results for all kind of simulation and estimation type for noisy measurement are listed in Table V.

Among the results (Figures 6(b), (d) and (f)), RMS errors yield a minimum value for linear type of simulation for all simulation function forms. Constant simulation yield the maximum value of RMS error, no matter which type were used for estimation. For the same simulation, the constant estimation has the worse RMS error and parabolic estimation have given us the better RMS than the linear and constant; but there is a very small difference between the linear and parabolic forms and regarding the CPU cost, we recommend the linear estimation (linear and parabolic forms need four and six parameters searching, respectively). Finally, the error comparison vs the time has been shown in Figure 7 for sensor Nos 2 and 3. Based on order of errors, excellent agreement has been achieved.

Table III. Best fitness and parameters with the same type of simulation and searching parameter for different runs based on exact measurements ($\sigma = 0$) which are shown in Figures 6(a), (c) and (e)

Measurements				Inverse analysis				
Simulation type	Run, generation	k_1	k_2	$-k_3 \times 10^5$	c_1	c_2	$-c_3 \times 10^4$	RMS error, fitness function
Constant simulation	1	14.100	–	–	448.000	–	–	1.28E-16
$k_1 = 14.1$ $c_1 = 448$	100							999.99
$n = 2$	2	14.100	–	–	448.000	–	–	2.05E-15
	100							999.95
	3	14.100	–	–	448.000	–	–	3.37E-17
	100							999.99
Linear simulation	1	14.100	0.01660	–	448.000	0.29100	–	3.03E-16
$k_1 = 14.1$ $c_1 = 448$	700							999.98
$k_2 = 0.0166$ $c_2 = 0.291$	2	14.100	0.01660	–	448.000	0.29100	–	2.72E-14
$n = 4$	700							999.84
	3	14.100	0.01660	–	448.000	0.29100	–	1.53E-10
	700							987.79
Parabolic simulation	1	14.099	0.01991	2.089	448.000	0.55010	5.597	3.30E-05
$k_1 = 14.1$ $c_1 = 448$	1,000							148.23
$k_2 = 0.020$ $c_2 = 0.5455$	2	14.100	0.02004	2.095	448.010	0.54960	5.610	4.33E-05
$k_3 = -0.000021$	1,000							131.88
$c_3 = -0.0005597$	3	14.096	0.02009	2.140	447.970	0.550100	5.580	4.79E-05
$n = 6$	1,000							126.25

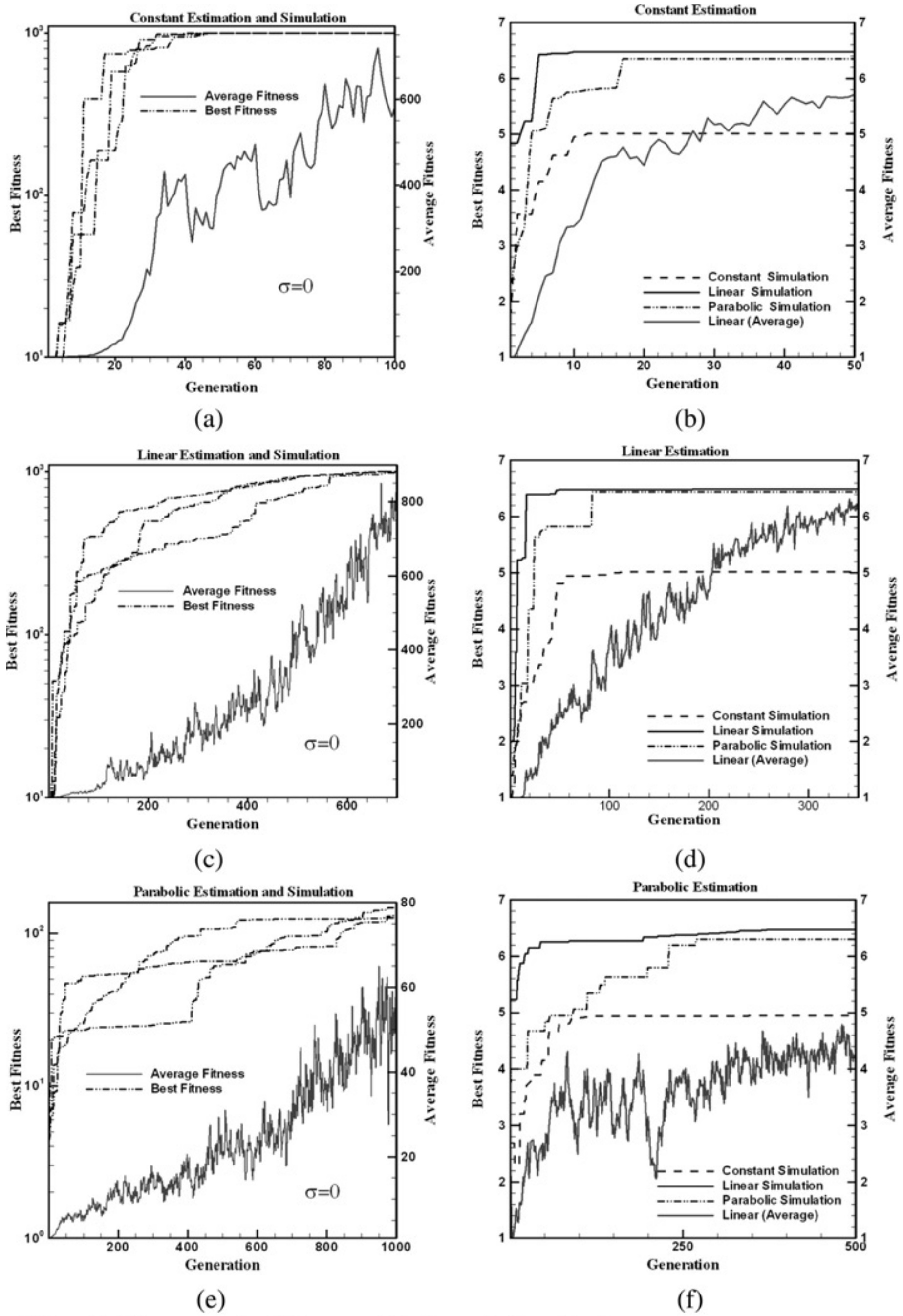


Figure 6.
Best and average fitness functions based on exact measurements (left, $\sigma = 0$) and noisy measurement (right, $\sigma = 0.01T_{max}$)

Notes: (a, b) constant, (c, d) linear and (e, f) parabolic estimation

8. Conclusion

An approach has been investigated to simultaneously estimate the temperature-dependent heat capacity and thermal conductivity using an inverse heat conduction method based on MEGA. Three cases of dependency for TDTPs have been investigated

Table IV. Best parameters with linear simulation and estimation parameter for different runs based on noisy measurements

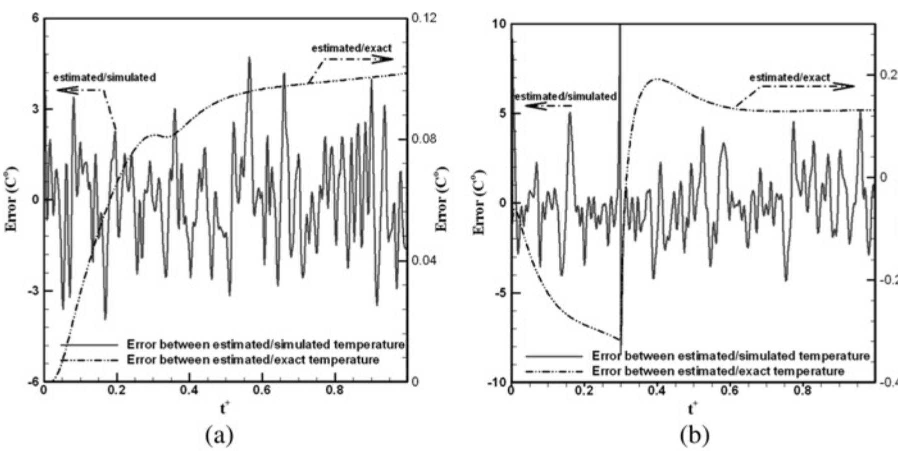
Measurements	Run	Inverse analysis k_1	k_2	c_1	c_2	RMS error
Linear simulation $\sigma = 0.01T_{max}$	1	14.1473	0.016458	447.025	0.30963	2.171E-02
	2	14.2425	0.016097	451.282	0.28509	2.228E-02
	3	14.1544	0.016506	449.681	0.27904	2.259E-02
	4	14.2156	0.016268	451.048	0.27968	2.289E-02
	5	14.0397	0.017031	443.901	0.31127	2.344E-02
	Mean	14.1599 ±0.1202	0.016472 ±0.0006	448.587 ±4.6865	0.29294 ±0.0183	2.258E-02 ±0.0009

Note: $\sigma = 0.01T_{max}$

Table V. Estimated parameters based on noisy measurements

Measurements Simulation	Type	Inverse analysis k_1	k_2	k_3	c_1	c_2	c_3	RMS error
Constant	Constant	14.1458	–	–	447.5618	–	–	0.039402
	Linear	14.0839	0.0001	–	446.4861	0.009651	–	0.036753
	Parabolic	13.9420	0.0017	−3.088E-06	445.9544	0.003864	−5.059E-05	0.037696
Linear	Constant	14.1624	–	–	447.9989	–	–	0.023509
	Linear	14.1599	0.016472	–	448.587	0.29294	–	0.022584
	Parabolic	13.9326	0.020037	−9.098E-06	441.4876	0.402980	−3.377E-04	0.022487
Parabolic	Constant	14.1293	–	–	447.8212	–	–	0.024443
	Linear	14.1689	0.019888	–	448.3175	0.549968	–	0.023779
	Parabolic	13.8521	0.022120	−2.490E-05	445.5987	0.508789	−3.727E-04	0.023637

Note: $\sigma = 0.01T_{max}$



Notes: (a) Sensor No. 2 and (b) sensor No. 3

Figure 7. Error comparison between estimated and simulated/exact temperatures with $\sigma = 0.01T_{max}$

and the results have been shown. Measurements are taken from sensors. The results show that the measurement errors do not considerably affect the accuracy of the estimates. Linear simulation yielded to the minimum RMS error and also it is good choice for estimation TDTPs. The proposed method provides a practical and confident prediction in simultaneously estimating the temperature-dependent heat capacity and thermal conductivity. This method is also applicable to other kinds of inverse heat transfer problems such as estimation of the directional thermo-physical properties, unknown heat flux estimation, inverse heat convection and radiation problems.

References

Alifanov, O.M. and Mikhailov, V.V. (1978), "Solution of the nonlinear inverse thermal conductivity problem by the iteration method", *Journal of Engineering Physics and Thermophysics*, Vol. 35 No. 6, pp. 1123-9.

Beck, J.V. and Al-Araji, S. (1978), "Investigation of a new simple transient method of thermal property measurement", *ASME Journal of Heat Transfer*, Vol. 96, pp. 59-64.

Beck, J.V. and Arnold, K.J. (1977), *Parameter Estimation in Engineering and Science*, Wiley, New York, NY.

Beck, J.V., Blackwell, B. and St-Clair, C.R. (1985), *Inverse Heat Conduction: Ill-posed Problems*, Wiley-Interscience, New York, NY.

Chen, H.T. and Lin, J.Y. (1998), "Simultaneous estimations of temperature dependent thermal conductivity and heat capacity", *International Journal of Heat and Mass Transfer*, Vol. 41 No. 14, pp. 2237-44.

Chen, T.F., Lin, S. and Wang, J.C.Y. (1997), "Determination of the boundary temperature errors by variation of two temperature sensor locations in a one-dimensional inverse heat conduction problem", *International Journal of Numerical Methods for Heat and Fluid Flow*, Vol. 7 No. 7, pp. 635-46.

Davis, L. (1991), *Handbook of Genetic Algorithms*, Van Nostrand Reinhold, New York, NY.

Doyle, J.F. (1995), "Determining the size and location of a crack in a frame structure", *Proceedings of the Seventh Inverse Problems in Engineering Seminar*, OSU, Columbus, OH.

Garcia, S. (1999), "Experimental design optimization and thermophysical parameter estimation of composite materials using genetic algorithms", PhD thesis, Virginia Polytechnic Institute and State University, Blacksburg, VA.

Gerald, C.F. and Wheatley, P.O. (1997), *Applied Numerical Analysis*, Addison-Wesley, Reading, MA.

Goldberg, D.E. (1989), *Genetic Algorithms in Search Optimization and Machine Learning*, Addison-Wesley, Reading, MA.

Huang, C.H. and Ozisik, M.N. (1991), "Direct integration approach for simultaneously estimating temperature-dependent thermal conductivity and heat capacity", *Numerical Heat Transfer, Part A*, Vol. 20, pp. 95-110.

Huang, C.H. and Yan, J.Y. (1995), "An inverse problem in simultaneously measuring temperature dependent thermal conductivity and heat capacity", *International Journal of Heat Mass Transfer*, Vol. 38, pp. 3433-41.

Huang, C.H., Yan, J.Y. and Chen, H.T. (1995), "Function estimation in predicting temperature-dependent thermal conductivity without internal measurements", *AIAA, Journal of Thermophysics and Heat Transfer*, Vol. 9 No. 4, pp. 667-73.

Imani, A., Ranjbar, A.A. and Esmkhani, M. (2006), "Simultaneous estimation of temperature-dependent thermal conductivity and heat capacity based on modified genetic algorithm", *Inverse Problems in Science and Engineering*, Vol. 14 No. 7, pp. 767-83.

Kim, S. (2001), "A simple direct estimation of temperature-dependent thermal conductivity with Kirchhoff transformation", *International Communications in Heat and Mass Transfer*, Vol. 28 No. 4, pp. 537-44.

Kim, S., Lee, K.J., Ko, Y.J., Kim, M.C. and Kim, K.Y. (2004), "Estimation of temperature-dependent thermal conductivity and heat capacity per unit volume with a simple integral approach", *International Journal of Heat and Mass Transfer*, Vol. 31 No. 7, pp. 981-90.

Kim, S.K. and Lee, W.I. (2002), "Inverse estimation of steady-state surface temperature on a three-dimensional body", *International Journal of Numerical Methods for Heat and Fluid Flow*, Vol. 12 No. 8, pp. 1032-50.

Kim, S.K., Jung, B.S., Kim, H.J. and Lee, W.I. (2003), "Inverse estimation of thermophysical properties for anisotropic composite", *Experimental Thermal and Fluid Science*, Vol. 27, pp. 697-704.

Martorano, M.A. and Capocchi, J.D.T. (2000), "Heat transfer coefficient at the metal-mould interface in the unidirectional solidification of Cu-8%Sn alloys", *International Journal of Heat and Mass Transfer*, Vol. 43, pp. 2541-52.

Michalewicz, Z. (1996), *Genetic Algorithms + Data Structure = Evolution Programs*, Springer, Berlin.

Ozisik, M.N. and Orlande, H.R.B. (2000), *Inverse Heat Transfer Fundamentals and Applications*, Taylor & Francis, New York, NY.

Ranjbar, A.A. and Mirsadeghi, M. (2007), "An inverse estimation of initial temperature profile in a polymer process", *Polymer Engineering and Science*, Vol. 48 No. 1, pp. 133-40.

Raudenský, M. (1993), "Heat transfer coefficient estimation by inverse conduction algorithm", *International Journal of Numerical Methods for Heat and Fluid Flow*, Vol. 3 No. 3, pp. 257-66.

Scarpa, F., Milano, G. and Pescetti, D. (1993), "Thermophysical conductivity estimation from transient data: Kalman versus Gauss approach", *Inverse Problems in Engineering: Theory and Practice*, ASME (Mech. Engineers), New York, NY, pp. 109-16.

Schaffer, J.D., Caruna, R.A., Eschelman, L.J. and Das, R.A. (1989), "Study of control parameters affecting online performance of genetic algorithms for function optimization" *Proceedings of the Third International Conference on Genetic Algorithms*, Morgan Kaufmann Publishers, San Francisco, CA, pp. 51-60.

Tervola, P. (1989), "A method to determine the thermal conductivity from measured temperature profiles Internat", *International Journal of Heat and Mass Transfer*, Vol. 32 No. 8, pp. 1425-30.

Venkatesan, G., Jin, G.P., Chyu, M.C., Zheng, J.X. and Chu, T.Y. (2001), "Measurement of thermophysical properties of polyurethane foam insulation during transient heating", *International Journal of Thermal Science*, Vol. 40, pp. 133-44.

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